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LATERAL TORSIONAL STABILITY OF MEMBERS WITH LATERAL RESTRAINTS AT VARIOUS LOCATIONS BETWEEN THE SUPPORTS

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ABSTRACT

This paper provides a general procedure to determine the elastic critical (Euler) lateral torsional buckling moment of a member with lateral restraints at various locations between the supports. The mathematical procedure to solve the eigenvalue problem is presented and some design examples are presented. The effectiveness of the locations on the member where the lateral restraints are applied, together with the effects of the flexibility of the restraints on the bearing capacity of the member is studied.

Key Words: *I-sections, lateral torsional buckling, bending, simulations, buckling*

1. INTRODUCTION

When designing members in bending, the designer has to pay much attention to the lateral torsional stability of the member. In modern codes, like the Eurocode 3 for Steel-structures [1], attention is given to the verification of the structural safety of members with respect to lateral torsional stability of members in bending. This is expressed in the so-called unity-check, where the ratio between the effects of the loading on the member and the capacity of the member, with respect to lateral torsional stability, needs to be smaller than unity or at maximum equal to unity to prove the stability of the member in bending. Determining the effects of the loading on the member is day-to-day business for the designer. However, determining the capacity of the member with respect to lateral torsional stability leads many times to a lot of work even in the case of determining this capacity via the elastic critical (Euler) lateral torsional buckling moment of the member. The elastic critical (Euler)

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lateral torsional buckling moment of the member is dependent on many parameters governing the phenomena. Many code writers consider the determination of the elastic critical (Euler) lateral torsional buckling moment of the member as a pure textbook related activity and from the code only a general reference to literature is given. Now here lies the problem for the designer. What literature is correct, which solution is applicable to the design situation at hand and what to do in cases the designer cannot find a sufficiently suitable solution for his design case.

2. PROGRAM DESCRIPTION

2.1 General

With Eurocode 3 the verification of the structural safety of members with respect to lateral torsional stability may be done by using the elastic critical (Euler) lateral torsional buckling moment of the member. This moment is defined as the maximum moment in the member due to the loading for which elastic critical instability occurs. The verification depends mainly on the solution of an eigenvalue problem. In practice this means that a multiplication factor for the loading on the member should be determined for which (Euler) lateral torsional instability occurs.

In some papers on stability the maximum (Euler) sagging moment is calculated. To gain a reliable verification rule this should be transferred to the maximum moment in the member.

2.2 Theoretical background

For the derivation of formulae to determine the theoretical elastic (Euler) lateral torsional buckling moment, several methods are suitable. One of the options is an energy method based on the Ritz method [2], [3]. According to this method the following equation holds:

$$\delta\Pi = \delta\Pi_e + \delta\Pi_g + \delta\Pi_p = 0 \tag{1}$$

with:

$$\delta\Pi_e = \int [EI_z\, v''\, \delta v'' + EI_w\, \varphi''\, \delta\varphi'' + GI_t\, \varphi'\, \delta\varphi')]\, dx \tag{2}$$

$$\delta\Pi_g = \int [M\, (v''\, \delta v' + \varphi\, \delta v'')]\, dx \tag{3}$$

$$\delta\Pi_p = - \int q\, \delta v\, dx \tag{4}$$

In equation (1) $\delta\Pi$ is the variation of the total energy Π (virtual work). The total variation of the energy related to lateral torsional buckling due to a disturbance of the state of equilibrium is equal to the virtual work $\delta\Pi_e$ due to bending out of plane, warping and torsion, the virtual work $\delta\Pi_g$ due to the geometrical first order non-linear effects and the virtual work $\delta\Pi_p$ due to the loading. The parameters v and φ are the out of plane displacements and the rotation around the longitudinal axis of the member, respectively. EI_z is the out of plane bending stiffness, EI_w is the warping stiffness and GI_t is the torsional stiffness of the beam.

Into equation (1) any arbitrary displacement field (displacements as well as rotations) can be substituted and this equation can subsequently be solved. Dependent on the displacement field chosen this results in a set of equations, which forms an eigenvalue problem. The quality of the solution depends heavily on the chosen displacement field. The better the chosen displacement field agrees with the real lateral torsional buckling mode the better the solution will be.

In the program for the displacements as well as for the rotations a sinus series is chosen. When sufficient terms of the sinus series are taken into account every possible mode for the displacement field can be approached accurately. In the program the number of terms of the sinus series can be set.

These sinus series are only used for the displacement field of the member. Using a sinus series for the loading on the member is not suitable, because too many terms would be necessary for an accurate description, while it can be done directly with an exact description. This direct way of describing the loading has as disadvantage that the equations become rather large and complex.

When the equations for the several terms are completely written, two matrices follow. The first is the linear stiffness matrix S_1. The second is the first order non-linear stiffness matrix S_2, which is dependent on the level of the loading. In this way the determination of the theoretical elastic (Euler) lateral torsional buckling moment is reduced to solving the following eigenvalue problem.

$$(S_1 + \lambda\, S_2)\, \underline{v} = \underline{0} \tag{5}$$

From the solution of equation (5) the critical loading and the deflected shape of the beam, at the moment lateral torsional buckling occurs, follow.

Because it is an eigenvalue problem, all the lateral torsional buckling loads and lateral torsional buckling modes are obtained. The number of modes is equal to the number of degrees of freedom of the chosen displacement field. The lowest positive eigenvalue produces the decisive theoretical elastic (Euler) lateral torsional buckling moment with the corresponding lateral torsional buckling mode.

In deriving the method the assumption that the shape of the cross section remains the same before and after lateral torsional buckling of the member occurs, is used. Further all classic assumptions for an Euler approach are used, such as perfect linear material behaviour, no residual stresses in the member, no initial deformations in the member, etc.

2.3 Relation between M_{cr} determined according to the numerical model and theoritical solutions

According to Eurocode 3, the theoretical solution M_{cr} can be described in case of double symmetrical cross sections and no end fixities at the supports, as:

$$M_{cr} = C_1 \frac{\pi^2 EI_z}{l_{LT}} \sqrt{\frac{I_w}{I_z} + \frac{l_{LT}^{\,2} GI_t}{\pi^2 EI_z} + (C_2 z_g)^2} - C_2 z_g \tag{6}$$

The C_1 and C_2 factors can be taken from literature for specific cases. So the elastic lateral torsional buckling moment is available only for those cases. The computer program described produces the elastic lateral torsional buckling moment for any double symmetric prismatic beam, loaded by any loading causing bending about the major axis and with lateral restraints at various locations.

The determination of C_1 goes as follows. Assume that the loading on the beam works at the centroidal axis. In such a situation the contribution of the C_2 factor, accounting for the effect of the loading point on the beam, equals zero. From the equation with only C_1 unknown, C_1 can be calculated. Now the loading can be placed at any point lower or higher than the centroid of gravity of the cross section and because C_1 is already known, from this one equation C_2 can be calculated.

In studying the relationship between the factor C_1 and the end moments, at the beam supports, an influence of the beam length is observed. In case that the end moments produce a constant moment in the beam ($\beta = M_1/M_2 = 1{,}0$) the influence of the beam length is absent. In case that the end moments produce a counter flexure in the beam ($\beta = M_1/M_2 = -1{,}0$) the influence of the beam length is about 10%. In figure 1 this influence is presented. In literature this spread in C_1-values is mostly motivated by the inaccuracy of the calculation model used. However from this study it is found that this spread is caused by the assumed constant ratio between torsion and warping in the formula for M_{cr}. For practical design the span to height ratio for a beam can be taken as 30.

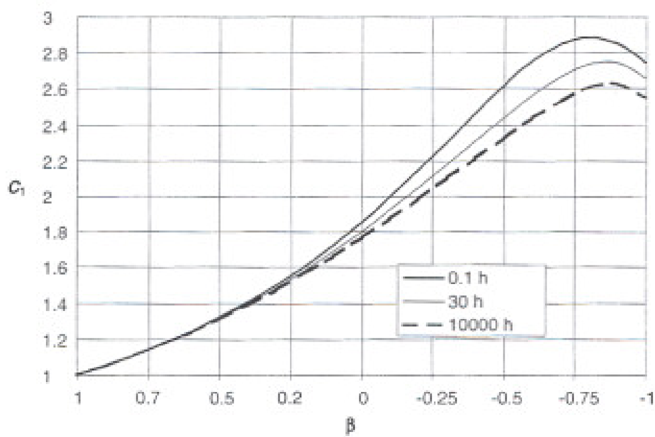

Fig. 1 - Spread in C_1-values

From calculations, with a broad range of thin walled as well as thick walled profiles, it is found that for general use of the C_2 factors, they need to be determined for the situation that the loading acts at the centroid of one of the flanges of the cross section.

3. BEAMS WITHOUT LATERAL RESTRAINTS BETWEEN THE SUPPORTS

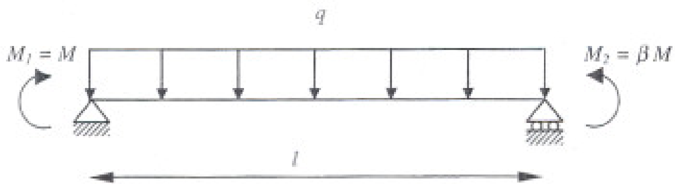

Fig. 2 - Beam with distributed loading and end moments

For the situation as shown in fig. 2 the C_1 value is calculated dependent on the factors β and B, where β is the ratio of the end moments at the supports of the beam and:

$$B = \frac{8M}{8|M| + ql^2} \tag{7}$$

The results are presented in Fig. 3. From Fig. 3 it can be observed that in the region of about β = -0,5 the shape of the C_1-surface is very steep. This region represents the transition of flange with the largest sideway displacement from the upper flange to the lower flange. This steep C_1 surface has a disadvantage. A small deviation in the end moments (hogging moments) of the beam causes a large deviation of the C_1-value and so of the elastic critical lateral torsional buckling moment. For real proportional loading, which mean that the end moment and the distributed loading have a full correlation, this phenomena is not very harmful. However, in design practice there is not a full correlation between those loading effects. Therefore the C_1-value in that region is limited to C_1 = 2,3. In determining the elastic critical lateral torsional buckling moment M_{cr} directly by means of an eigenvalue calculation, this phenomenon is not easily recognized and can lead to unsafe design. Because the C_2 factor is related to the C_1 factor, by limiting the C_1 factor the C_2 factor is limited consequently.

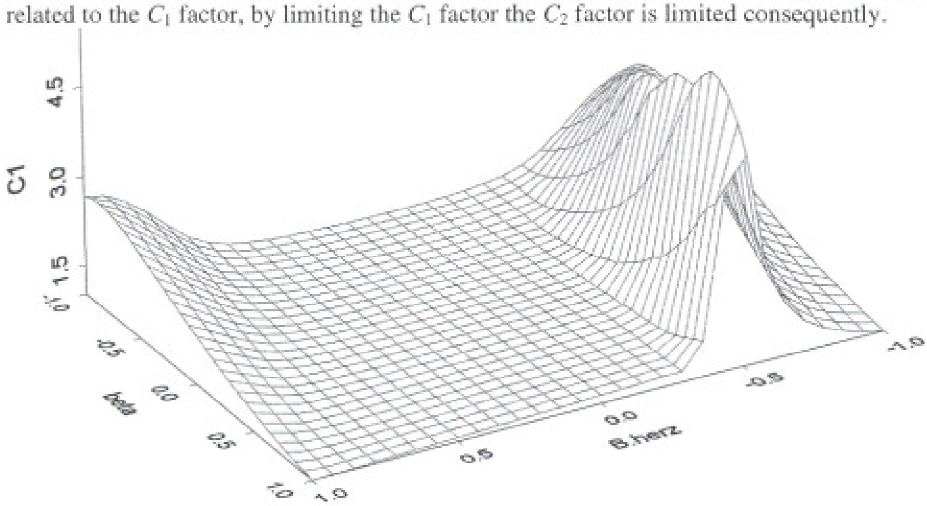

Fig. 3 - C_1 factor as function of β and B

4. BEAMS WITH LATERAL RESTRAINTS

4.1 General

Eurocode 3 only provides rules for beams, where the supports form fork conditions. No provisions are given for intermediate lateral restraints.

Based on the calculation model described in chapter 2, lateral restraints at various locations between the supports can be taken into account. In the next chapters some specific cases are described.

4.2 Beam with uniform moment distribution

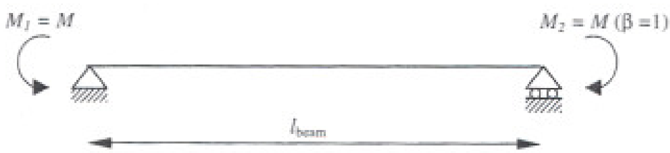

Fig. 4 - Beam subjected to uniform moment

For a beam subjected to a moment distribution described by β=1, see Fig. 4, supported at the ends with forks, and no intermediate lateral restraints, the elastic critical lateral torsional buckling moment and the corresponding eigenmode is given in Fig. 5.

$$M_{cr} = \frac{\pi^2 EI_z}{l_{beam}} \sqrt{\frac{I_w}{I_z} + \frac{l_{beam}{}^2 GI_t}{\pi^2 EI_z}}$$

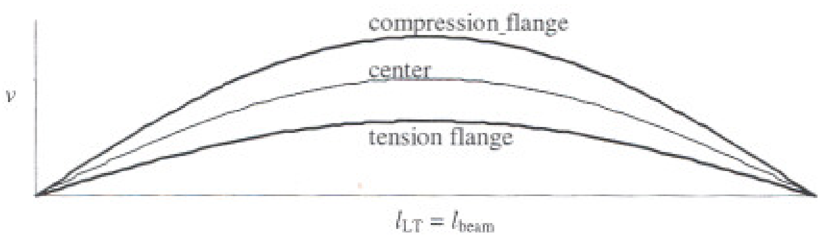

Fig. 5 - Beam subjected to constant moment (β=1) without intermediate lateral restraints

When an intermediate lateral restraint is applied at midspan, irrelevant on the position above the centroid, the elastic critical lateral torsional buckling moment and the corresponding eigenmode is given in Fig. 6.

$$M_{cr} = \frac{\pi^2 EI_z}{0.5 l_{beam}} \sqrt{\frac{I_w}{I_z} + \frac{(0.5 l_{beam})^2 GI_t}{\pi^2 EI_z}}$$

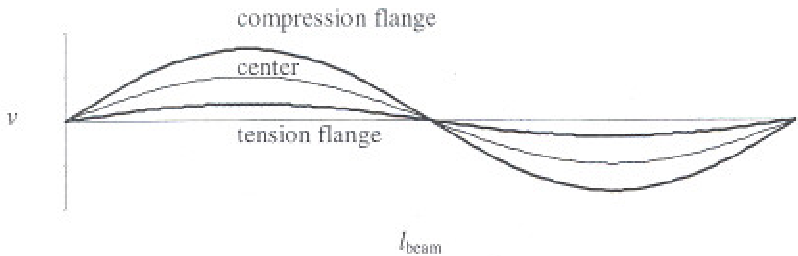

Fig. 6 - Beam subjected to constant moment (β=1) with intermediate lateral restraint at midspan

For a beam with constant moment, fork supports at both ends and with half the length of the previous beam, the elastic critical lateral torsional buckling moment appears to be the same as the previous beam. From this it can be concluded that in this case the lateral restraint at midspan can be considered as being a fork.

The influence of the position of the restraint at midspan is investigated in Fig. 7. From this figure it can be seen that for every position of the restraint above -0.6 $(h-t_f)/2$ the restraint acts as a fork. However, from Fig. 8 it can be seen that the required critical spring stiffness depends heavily on the position of the spring.

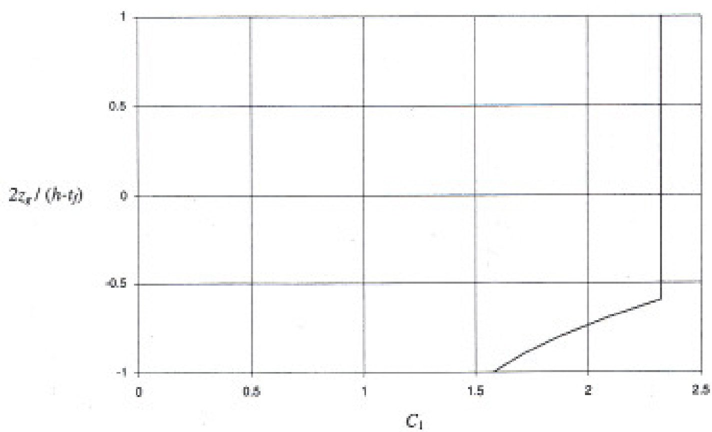

Fig. 7 - Influence of the position of the restraint

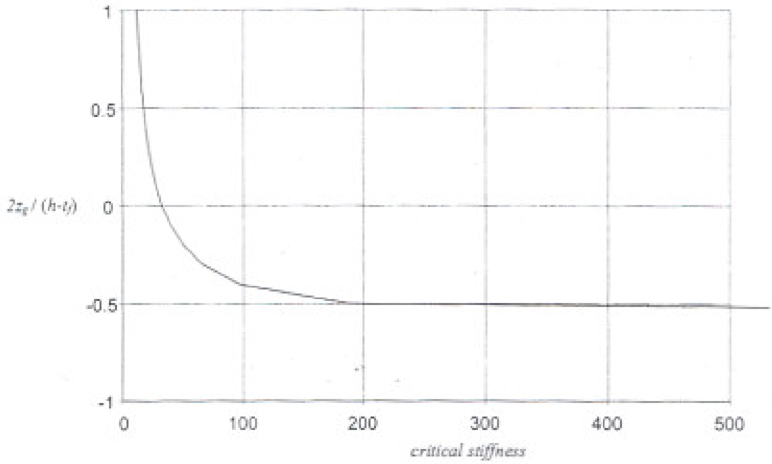

Fig. 8 - Influence of the position of the restraint on the critical stiffness [N/mm] of a lateral restraint at midspan of an IPE200 beam with a span equals 30 times the beam height

4.3 Beam with non-uniform moment distribution

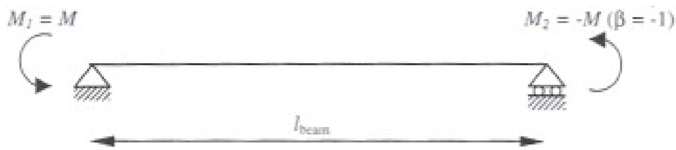

Fig. 9 - Beam subjected to a moment distribution described with $\beta = -1$

For a beam subjected to a moment distribution described by $\beta = -1$, supported at the ends with forks and no intermediate lateral restraints, the elastic critical lateral torsional buckling moment and the corresponding eigenmode is given in Fig. 10.

$$M_{cr} = 2.65 \frac{\pi^2 EI_z}{l_{beam}} \sqrt{\frac{I_w}{I_z} + \frac{l_{beam}^2 GI_t}{\pi^2 EI_z}}$$

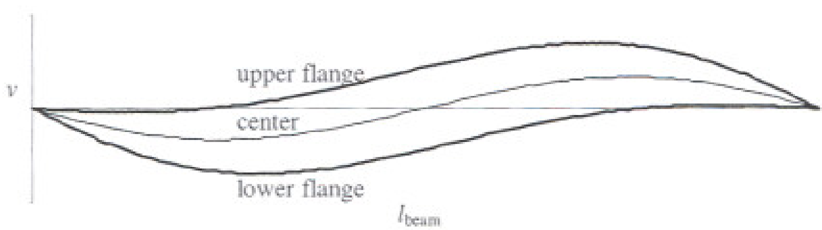

Fig. 10 - Beam subjected to end moments ($\beta = -1$) without intermediate lateral restraints

In Fig. 10 it can be seen that the midspan cross section does rotate along the beam axis but does not undergo lateral displacements at the centroid. From this it can be concluded that a lateral restraint at the centroid and at midspan is ineffective. From calculations it also appears that a lateral restraint at the upper or lower flange at midspan is also ineffective in this case.

In design practice it is convenient to verify beams with lateral restraints at various locations between the supports by looking to the individual parts of the beam between a fork and a lateral restraint and to the individual parts of the beam between lateral restraints. See Fig. 11.

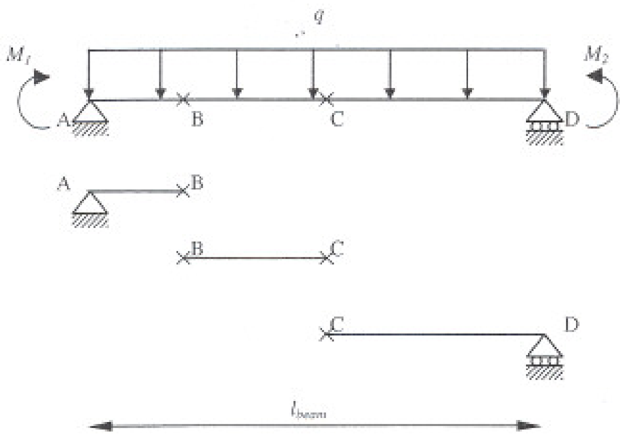

Fig. 11 - Subdivision of a beam for code checking

For the individual beam parts AB, BC and CD the own non-uniform moment distributions can be determined. To determine the elastic critical lateral torsional buckling moment for each part the length of that part needs adjustment by multiplying that length by a factor, which is composed from a parameter study using the numerical model.

$$l_{LT} = (1,4 - 0,8\ \beta)\ l_{beam\ part} \qquad (8)$$

where:

$$1,0 \leq l_{LT}\ /\ l_{beam\ part} \leq 1,4 \qquad (9)$$

This procedure is only valid under the condition that the lateral restraints are attached to the beam at that location, which would undergo the largest displacement in case that the lateral support would not be present.

5. CONCLUSIONS

The numerical model is a powerful tool to calculate the elastic critical lateral torsional buckling moment for double symmetric, prismatic beams with fork conditions at both ends and lateral restraints at various locations between the supports.

Using such a tool or similar tools based on FEM need to be done with care in regions where the elastic critical lateral torsional buckling moment is very sensitive to small changes in de moment distribution.

From the analysis with the presented numerical model it can be concluded that the use of lateral restraints is highly effective even in cases where the location is not optimal. This leads to the observation that many structural elements that might be present for other reasons are relative effective lateral restraints even without special measures to transform them to optimal lateral restraints. This contributes highly to designing economic steel structures.

Still research need to be done for studying the lateral torsional stability of cantilever beams with or without lateral restraints.

7. REFERENCES

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