

## **YIELD LIMIT VS. BEHAVIOUR FACTOR IN SEISMIC DESIGN**

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**Abstract:** According to the regulations in EC8, two identical structures would have the same behaviour factor, even if one is made out of S235 and one out of S355. Thus they would have an equal calculatory design base shear. However, the structure made out of S355 will develop a larger amount of storey drift before going plastic under cyclic ground acceleration, which results in considerably higher reaction forces. This is a lack of regulation, since S355 could be seen as S235 with excessive overstrength. This article reports on a study where the dynamic behaviour of simple structures made of different materials is investigated by nonlinear time history analyses. Design recommendations for the amendment of EC8 are proposed.

### **1 Introduction**

When designing steel structures according to EC8 [1], the behaviour factor is accounting for the ability of the structure to dissipate energy by yielding. If the designer opts for low-dissipative structural behaviour (predominantly elastic design), the behaviour factor is limited to 1.5, thus allowing for plastic action in members and joints, which is inherent in the structure even without special consideration in design and construction.

If the designer opts for dissipative design, behaviour factors up to 8 are enabled by proper choice of cross sectional class, joint detailing and management of possible overstrength of the materials used in fabrication.

According to the regulations in EC8, two identical structures would have the same behaviour factor, even if one is made out of S235 and one out of S355. Thus they would have an equal calculatory design base shear. However, when both are subjected to the same cyclic ground acceleration, the structure made out of S355 will develop a larger amount of storey drift before going plastic, which results in considerably higher reaction forces. This can be shown by means of non-linear time history analyses.

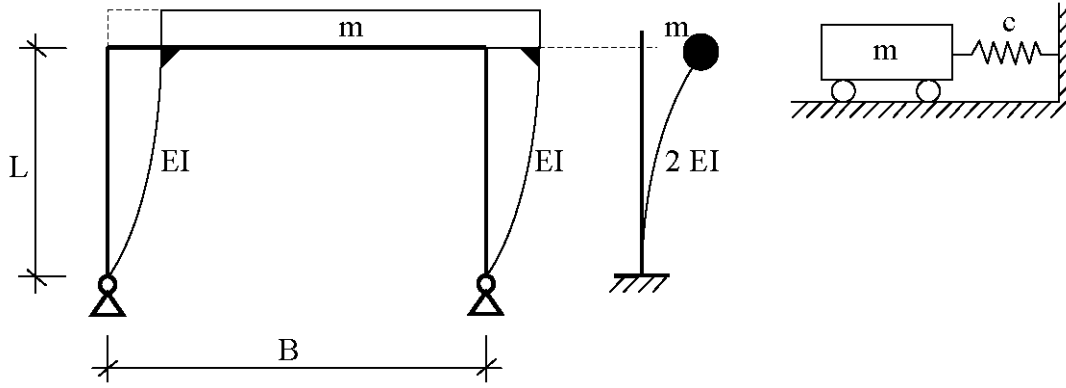
### **2 Typical sway frame configuration**

#### **2.1 Properties**

We use the following example for the numerical calculations:

Single storey two column plane sway frame with pinned column bases as shown in Fig. 1. The analysed frame has a width of 5 m, a height of 3 m and columns of cross section class 1, the head mass is assumed to be 72,000 kgs. The beam is of infinite stiffness, the columns are made up of sections HEA240 which are bent about their strong axes, thus giving a linear restoring stiffness of

$$c = 2 * 3 * EI / L^3 = 6 * 2.1 * 10^5 \text{ MPa} * 7760 \text{ cm}^4 / (3.0 \text{ m})^3 = 3.62 * 10^6 \text{ N/m} \quad (1)$$



**Fig. 1:** Dynamic model of a single storey frame  
left: two column frame; middle: SDOF cantilever; right: mass and spring oscillator

The elastic and full plastic moment respectively of a column HEA240-S235 is given by

$$M_{el,lim} = W * f_y / 1.1 = 675 \text{ cm}^3 * 235 \text{ Mpa} / 1.1 = 144 \text{ kNm} \quad (2)$$

$$M_{Rd} = 1.14 * W * f_y / 1.1 = 1.14 * 675 \text{ cm}^3 * 235 \text{ Mpa} / 1.1 = 164 \text{ kNm} \quad (3)$$

According to some tables the full plastic moment amounts to

$$M_{Rd} = 159 \text{ kNm} \quad (4)$$

which will be used for further calculations.

The associated values for the base  $F_b$  shear are

$$F_{b,el,R} = M_{el,lim} / H = 144 \text{ kNm} / 3.0 \text{ m} = 48 \text{ kN} \quad (5)$$

$$F_{b,pl,R} = M_{Rd} / H = 159 \text{ kNm} / 3.0 \text{ m} = 53 \text{ kN} \quad (6)$$

per column. The elastic limit storey drift is given by

$$x_{el,lim} = 2 * F_{b,el,R} / c = 2 * 48 \text{ kN} / 3.62 * 10^6 \text{ N/m} = 26.5 \text{ mm} \quad (7)$$

## 2.2 Seismic design

If we assume a soil factor of  $S = 1.5$  and a ground acceleration of  $a_{gR} = 1.6 \text{ m/s}^2$ , the driving acceleration amplitude for time-history analysis will be

$$a_{dyn} = a_{gR} * S = 1.6 \text{ m/s}^2 * 1.5 = 2.4 \text{ m/s}^2 \quad (8)$$

According to EC8 eq. 3.3 [1] the plateau of the elastic response spectrum is given by

$$S_e(T) = a_g * S * \eta * 2.5 \quad (9)$$

where

$$\eta = \sqrt{\frac{10\%}{5\% + \zeta}} \geq 0.55 \quad (10)$$

is a correction factor according to EC8 eq. 3.6 [1], which is equal to unity if the structural (viscous) damping  $\zeta = 5\%$ .

The plateau of the design response spectrum is given by

$$S_d(T) = a_g * S * 2.5 / q \quad (11)$$

according to EC8 eq. 3.14 [1].  $q \geq 1$  is the behaviour factor which relates the response of a purely elastic structure with 5% viscous damping to the response of a structure which

exhibits energy dissipation due to plasticity (EC8 3.2.2.5 (3) [1]). This paragraph claims explicitly that damping other than 5 % is covered by  $q$  as well. We will give some remarks on this in the section conclusions.

Increasing behaviour factors  $q$  indicate increasing potential to activate plastic action in the structure. As can be seen from the equations above, the elastic response and the design response are identical if  $q = 1$ . Since this procedure is meant to be used for a linear calculation of the base shear, it is implied that the horizontal displacements (storey drift) are also related to  $q$  in a linear proportion.

Employing the simplified lateral force method according to EC8 4.3.3.1 with eqs. 4.5 and 3.14 [1] we receive for the above data a total base shear of

$$F_b = m * \lambda * \gamma_I * a_{gR} * S * 2.5 / q \quad (12)$$

for  $q = 1.5$ :  $F_b = 72,000 \text{ kgs} * 1.0 * 1.0 * 1.6 \text{ m/s}^2 * 1.5 * 2.5 / 1.5 = 288 \text{ kN}$   
 for  $q = 4.0$ :  $F_b = 72,000 \text{ kgs} * 1.0 * 1.0 * 1.6 \text{ m/s}^2 * 1.5 * 2.5 / 4.0 = 108 \text{ kN}$

Comparing this with the above data we see that a design according to ductility class low (DCL) and a behaviour factor of  $q = 1.5$  leads to a utilisation of

$$\eta = F_b / 2F_{b,pl} = 288 \text{ kN} / 2 * 53 \text{ kN} = 2.72 \quad (13)$$

whereas using ductility class high (DCH) and a behaviour factor  $q = 4$  results in a utilisation of

$$\eta = F_b / 2F_{b,pl} = 108 \text{ kN} / 2 * 53 \text{ kN} = 1.02 \quad (14)$$

### 3 Dynamic model

#### 3.1 Set up

For this study we use a SDOF (single degree of freedom) oscillator which represents a one-storey frame. The stiffness of the oscillator can be switched by a factor of 1 to 0 compared to the original stiffness. The switching point can be associated to an arbitrary storey drift, so we can simulate plasticity after a certain amount of horizontal displacement.

Different habits are known to be used within the numerical simulating community, such as  $E/1000$ , 'zero' for ideal plastic behaviour and others.

We prefer to use a value which is based on the actual stress-strain-curve of the material. For S235 we use 235 MPa at zero plastic strain and the tensile strength 360 MPa at 20 % plastic strain. With this assumption we receive a secondary stiffness of

$$\Delta\sigma / \Delta\varepsilon = (360 \text{ MPa} - 235 \text{ MPa}) / 0.2 = 625 \text{ MPa} \quad (15)$$

which corresponds to 1/336 of Young's modulus. Likewise we receive 1/271 for S355. However, these differences do not influence the results very much. An internal study on the effect of the switch factor shows that the response of the structure does not significantly alter for switch factors smaller than 0.5.

The governing equations are evaluated for an individual time step, using initial displacement, velocity and acceleration. During the time step, a new displacement and a new velocity are calculated with the initial conditions where an external sinusoidal drive (excitation) is input. Equilibrium of forces is evaluated due to elastic or plastic restraint and damping, the residuum of which gives the acceleration at the end of the actual time step. The quantities from the end of the  $i^{\text{th}}$  time step are then used as initial conditions of the  $(i+1)^{\text{th}}$  time step. This procedure is robust against diverging, if the time increments are remaining small enough, e.g. 20-50 per driving period.

Simplifying, a sinusoidal force had been used in previous studies (Knoedel 2011 [2], Knoedel/Hrabowski 2011 [3]). This time a sinusoidal base excitation is used as drive.

### 3.2 Verification

With a mass of  $m = 72$  tons carried by the beam (including self weight of the structure) the lowest angular frequency of the sway motion amounts to

$$\omega = \sqrt{\frac{c}{m}} = \sqrt{\frac{3.62 * 10^6 \text{ N/m}}{72000 \text{ kgs}}} = 7.09 \frac{\text{rad}}{\text{s}} \quad (16)$$

corresponding to an eigenfrequency of

$$f = \omega / 2\pi = 7.09 \text{ rad/s} / 2\pi = 1.13 \text{ Hz} \quad (17)$$

This frequency is used for the base excitation, but we allow for a detuning of

$$k = \sqrt{1 - D^2} \quad (18)$$

which accounts for the reduction of the eigenfrequency due to the damping ratio  $D$ .

With a purely elastic system under this excitation the response amplitude  $x_{\text{response}}$  should converge at

$$x_{\text{response}} = x_{\text{base}} * \frac{1}{2D} = x_{\text{base}} * \frac{\pi}{\delta} \quad (19)$$

where

$$\delta = 2\pi D \quad (20)$$

is the logarithmic decrement and  $x_{\text{base}}$  the driving amplitude at the column base. NB: with base excited systems the response amplitude denotes the relative displacement between base and head of the frame.

With the assumption of EC8 [1] – that an ordinary building structure should have a damping ratio of  $D = 0,05$  – the response amplitude  $x_{\text{response}}$  should be subjected to an amplification of

$$1 / 2D = 1 / 0.10 = 10 \quad (21)$$

compared to the input base excitation amplitude  $x_{\text{base}}$ .

### 3.3 Drive

A previous study of Knoedel (2011) [2] showed that it is extremely unfavourable to excite the structure in its critical frequency until steady state amplitudes are developed. This holds especially for steel structures which might have a damping ratio as low as  $\delta = 0.015$  or  $D = 0.0024$  (Knödel/Heß 2011 [4]).

Therefore, we had a closer look at the characteristics of European seismic events given by Bachmann (2002) [5], which are not repeated here due to lack of space but can be accessed by Knödel 2010 [6]. Interestingly there are only some 5 to 7 cycle periods with major peaks of ground acceleration. Subsequently, the responses of a 1-Hz-oscillator and a 3-Hz-oscillator did not show more than 5 to 7 cycles of large amplitudes before decaying again.

Using this information a restriction of the drive to 10 full cycles seems to be sufficiently unfavourable for the present simulation. However, in EC8 3.2.3.1.2 (3) [1] it is demanded that the steady state acceleration should last for at least 10 seconds, which gives 12 full cycles for our chosen example.

Instead of using a harmonic base acceleration with an amplitude of

$$a_{\text{base}} = a_{\text{gR}} * S = 1.6 \text{ m/s}^2 * 1.5 = 2.4 \text{ m/s}^2 \quad (22)$$

we are using a harmonic base displacement with an amplitude of

$$x_{\text{base}} = a_{\text{base}} / \omega^2 = 2.4 \text{ m/s}^2 / (7.09 \text{ rad/s})^2 = 47.7 \text{ mm} \quad (23)$$

### 3.4 System response

In the following we compare different steel grades both 'plain' and with design-overstrength values given by the overstrength factor recommended by EC8 6.2 (4)

$$\gamma_{ov} = 1.25 \quad (24)$$

and condition EC8 6.2 (3) a) [1]

$$f_{y,max} \leq 1.1 * \gamma_{ov} * f_y = 1.1 * 1.25 * f_y = 1.38 f_y \quad (25)$$

In a first run the plastic limit load has been set to infinity, so the system remains purely elastic. The system's response shows unlimited growth of displacements as would be expected of an undamped system (see Table 1).

In EC8 [1] structural systems are assumed to have a viscous damping ratio of  $\zeta = 0.05$ . The actual maximum amplitude of a system with such damping is 468 mm, see Fig. 2.

With additional plastic hinges, which are determined by the fully plastic moment of a section HEA240–S235 at the column-bar-joints, the displacements of the plastic system are limited to about 35 mm, with a first peak reaching up to 75 mm. After some quasi chaotic behaviour a steady state phase-shift of  $180^\circ$  can be observed, see Fig. 3.

Another system has been set up where the elastic limit load is increased by 50 %, which corresponds to HEA240–S355. This is supposed to simulate different plastic response of bracing elements within the same structure or unintended overstrength. The displacements are slightly higher, as could have been expected, see Fig. 4.

The dynamic responses of the different systems investigated are summarized in Table 1.

The effective behaviour factor is per definition (EC8 3.2.2.5 (3) [1])

$$q_{eff} = x_{response,elastic} / x_{response,plastic} \quad (26)$$

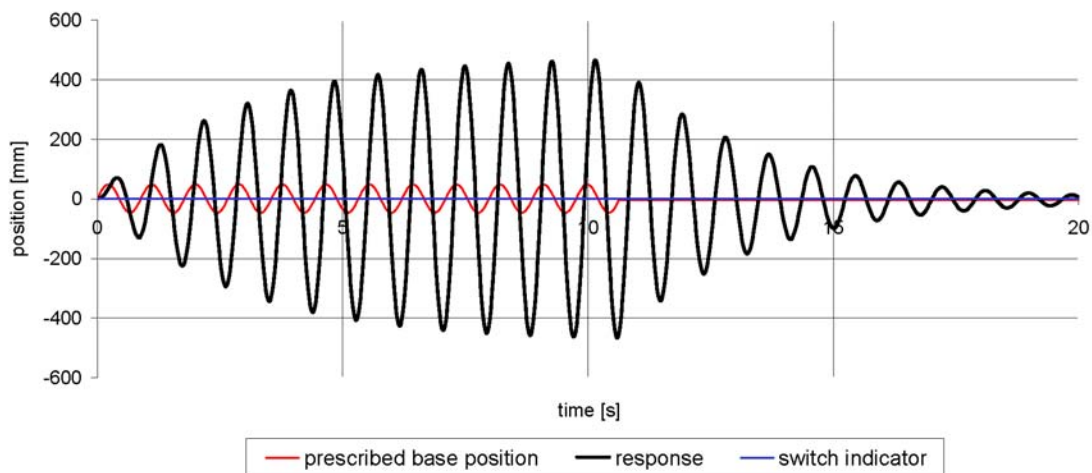
In the last but one column of Table 1 the maximum amplitude of the different systems investigated is divided by the elastic limit storey drift of 26.5 mm.

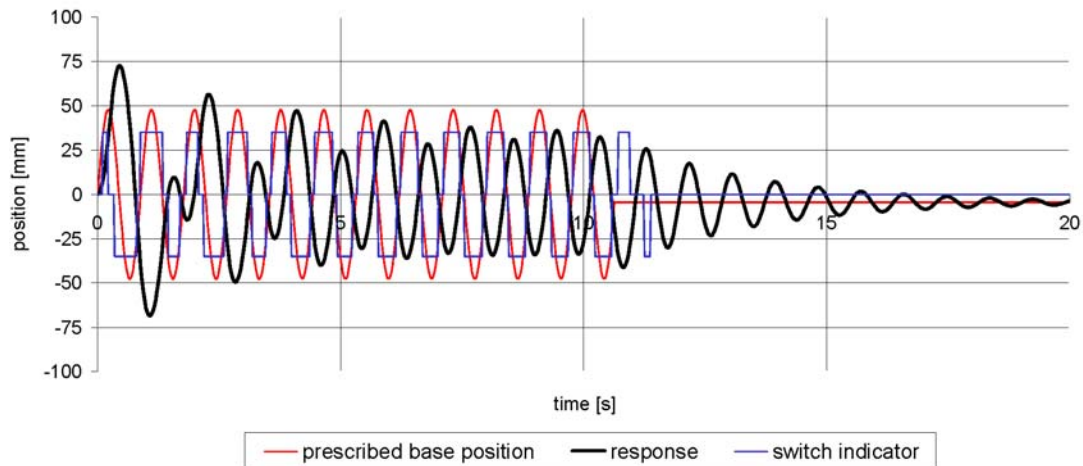
Comparing the peaks of the two systems with different elastic limit loads, we can see that the peaks are of different size and – apart from the first two peaks – do not occur simultaneously.

In general, it is a difficult question to compare different non-linear systems. From a structural mechanics point of view they are just different. Thus it might be more sensible to regard both systems as completely different dynamic systems with completely different response. From an engineering point of view you would want to compare them in order to understand their common features even if you know that this is based on very rough simplifications.

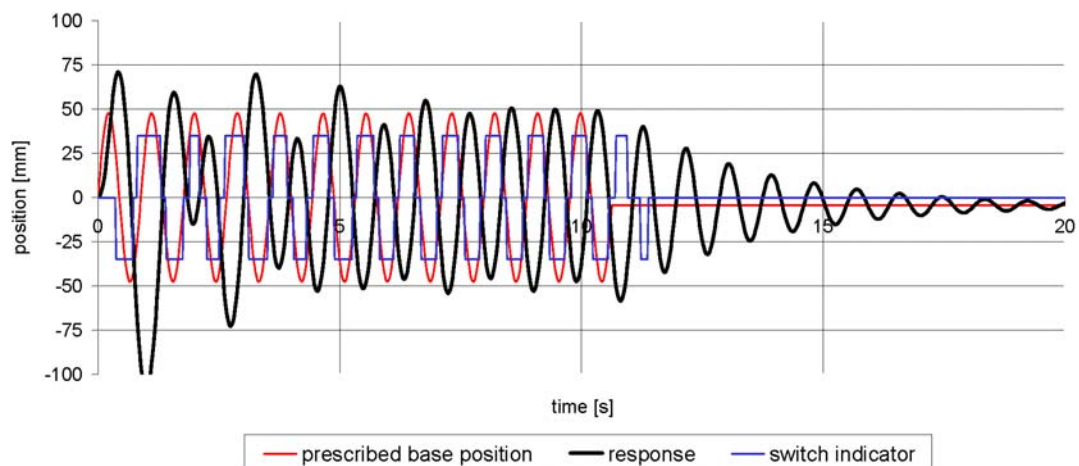
**Table 1:** System response of a single storey frame with limited drive

System	Normalised elastic limit load [%]	Maximum positive amplitude [mm]	Maximum negative amplitude [mm]	Elastic limit storey drift ratio	Effective behaviour factor	
Elastic undamped	$\infty$	1800	1800	68	–	
Elastic damped	$\infty$	466	468	18	1.0	
Plastic damped	S235	100	72	68	2.7	6.5
	S235 + overstrength	138	72	101	3.8	4.6
	S275	117	73	85	3.2	5.5
	S275 + overstrength	161	79	114	4.3	4.1
	S355	150	71	108	4.1	4.3
	S355 + overstrength	207	115	128	4.8	3.7
	S460	196	105	126	4.8	3.7
	S460 + overstrength	270	156	134	5.9	3.0
	S690	294	165	134	8.8	2.8
	S690 + overstrength	405	182	200	7.5	2.3
	S960	409	183	201	7.6	2.3
	S960 + overstrength	564	246	234	9.3	1.9

**Fig. 2:** Time-history plot of sway displacements, damped elastic system base excitation amplitude 47.7 mm



**Fig. 3:** Time-history plot of sway displacements, damped plastic system base excitation amplitude 47.7 mm

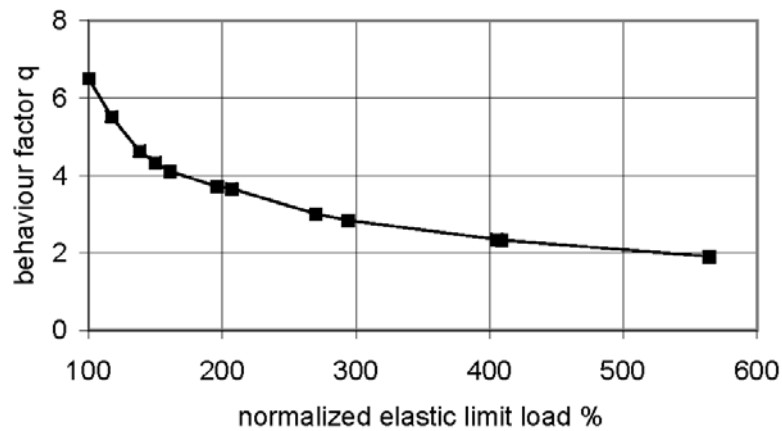


**Fig. 4:** Time-history plot of sway displacements, damped plastic system elastic limit load increased by 1.5, base excitation amplitude 47.7 mm

## 4 Conclusions and remarks

1. As we have, the lower value for the effective behaviour factor for a higher elastic limit load of the same system confirms that the use of steel grade S355 instead of S235 leads to higher seismic loads in an otherwise identical situation. The relation of the behaviour factors and the associated normalized elastic limit loads is given in Fig. 5.
2. This lack of regulation with regard to the yield limits of different structural materials seems to contradict the strict rules concerning overstrength, since S355 could be seen as S235 with excessive overstrength.
3. It might be expected that the loss of stiffness due to plastic behaviour prolongs the vibration period and at the same time the eigenfrequency becomes lower. However, the time-history plots in Fig. 3 and Fig. 4 show that the systems' response is perfectly synchronised to the drive (compare Knoedel 2011 [7]). An explanation can be given with respect to the intensity of plastic behaviour: with a driving amplitude of 47.7 mm and an elastic limit storey drift of 26.5 mm it is clear that the structure goes plastic within each cycle. This is indicated by rectangular blocks ('switch indicator') in the

above time history plots. Thus, while having lost a part of their stiffness, the structure can re-adjust to the drive during each half-wave of the cycle.



**Fig. 5:** Decreasing behaviour factors  $q$  with increasing material strength

4. The assumption in EC8 3.2.2.5 (3) [1] that the behaviour factor  $q$  covers damping ratios smaller than 5 % is doubtable from our point of view. As we saw above in Table 1, the response amplitude with a limited drive and  $\zeta = 0.05$  is some 468 mm. With a logarithmic decrement of  $\delta = 0.015$  (or  $D = 0.0024$ ), which is a familiar number for certain steel chimneys, this response amplitude would be as much as 1600 mm, three times as much. How shall this be covered for a structural situation with a behaviour factor close to 1? This topic has been discussed elsewhere, see Knödel/Heß (2011 [4]).
5. When we look at the above time history plots and Fig. 5, we can see that structures from higher strength materials exhibit bigger response amplitudes because they go plastic after bigger displacements and thus can absorb more from the offered energy. At the same time materials of higher strength mean that the utilisation of the structure in the cross-section check is low. So the problem does not lie in the use of higher strength materials but in low utilisation, which means that the structure is subjected to wider displacements before going plastic. On this basis we suggest an amendment for EC8 [1] in the next section.
6. Just before finishing this paper, the authors noticed a very interesting contribution by Brescia (2008 [8]), where the effect of overstrength on sufficient rotation capacity is pointed out.

## 5 Proposed amendment for EC8

In Fig. 6 we plotted the data of Fig. 5, but we normalized the behaviour factor to its maximum value. Additionally we changed the abscissa from normalized elastic limit load to utilisation. We can see that the effective behaviour factors are roughly proportional with decreasing utilisation.

Therefore, we suggest a modified behaviour factor with steel structures which is given by

$$q_{\text{mod}} = k_q * q \quad (27)$$

The number of  $k_q$  is identical to the utilisation of the dissipative structural parts.

To keep this new rule reasonable it could be added that this condition does not need to be employed as long as the utilisation is higher than 0.7.



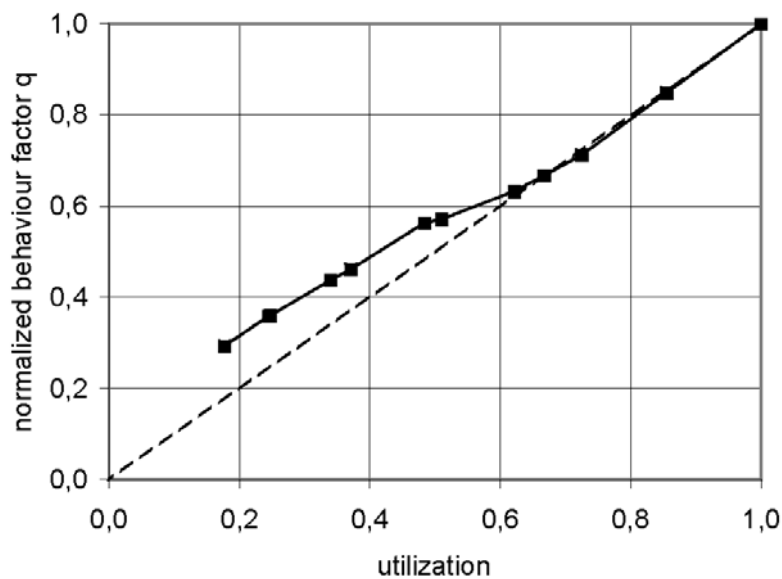


Fig. 6: Effective behaviour factor  $q$  with small utilisation

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