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SEISMIC ANALYSIS OF LIQUID STORAGE TANKS

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Abstract (Tank design for seismic loading): In the event of strong earthquakes, it is important that the structural integrity of tanks containing liquids is maintained in order to not jeopardize the population's supply with essential goods. The continuous operation of tanks after strong earthquakes requires safe and effective design rules. The overall seismic behaviour of tanks is, however, quite complex, since the dynamic interaction effects between tank wall and liquid must be considered. The interaction can be simplified with the concept of generalized single-degree-of-systems representing the convective, rigid impulsive and flexible impulsive vibration modes of tank and liquid. This concept is well accepted for anchored tanks with a fix connection to a rigid foundation. This paper presents the state of the art of tank design with special focus on the practicability of the available design rules. Analytical and numerical calculation approaches are compared on the example of a typical tank geometry, taken the relevant interaction effects into account.

Key words: liquid filled tank, seismic design, impulsive vibration mode, critical facilities

INTRODUCTION

Recent earthquake events showed that heavy seismic damages of tanks may lead to environmental hazards, fire following earthquakes and temporary loss of essential facilities. The vibration of liquid filled tanks subject to seismic loading depends on the inertia of the liquid and on the interaction effects between the liquid and the tank shell. Different calculation methods are available for describing the vibration behaviour and the earthquake loads. These methods are either quite simple (Housner, 1963) or very accurate but complex (Fischer et al., 1991). Therefore a well comprehensive and feasible method is needed, that provides realistic results for the seismic behaviour of tanks with an acceptable computing time. The following considerations apply to cylindrical, anchored tanks with a fix connection to a rigid foundation.

SEISMICALLY INDUCED LOAD COMPONENTS OF LIQUID FILLED TANKS

The seismic loads acting on wall and bottom of cylindrical tanks (Figure 1) can be divided into the following components (Meskouris et al., 2010):

- the convective load component; the fluid vibration in the rigid tank (sloshing),
- the impulsive rigid load component; caused by the inertia of the liquid, if the rigid tank moves together with the foundation,
- the impulsive flexible load component; representing the combined vibration of the flexible tank shell (e.g. steel tanks) with the liquid.

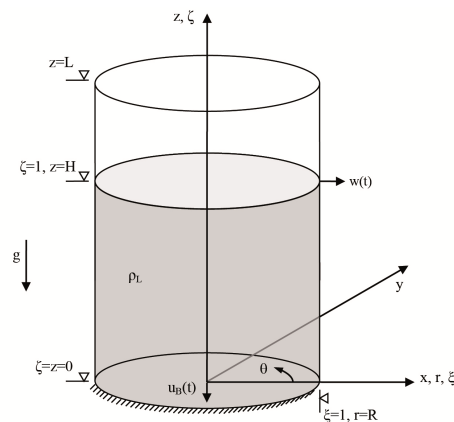


Fig. 1: Cylindrical tank

Convective pressure

Figure 2 shows the mode of vibration and the pressure distribution corresponding to the convective pressure component.

The pressure distribution is defined as:

$$p_k(\xi, \zeta, \theta, t) = \sum_{n=1}^{\infty} \frac{2 \cdot R \cdot \rho_L}{(\lambda_n^2 - 1)} \left[\frac{J_1(\lambda_n \cdot \xi)}{J_1(\lambda_n)} \right] \left[\frac{\cosh(\lambda_n \cdot \gamma \cdot \zeta)}{\cosh(\lambda_n \cdot \gamma)} \right] \cdot [\cos(\theta)] \cdot [a_{kn}(t) \cdot \Gamma_{kn}] \quad (1)$$

with

- p_k convective pressure component due to horizontal excitation
 n summation index; number of considered sloshing modes (here: $n = 1$)
 R inner tank radius
 ρ_L liquid density



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J_1 first order Bessel function:

$$J_1(\lambda_n \cdot \xi) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k! \cdot \Gamma(1+k+1)} \cdot \left(\frac{\lambda_n \cdot \xi}{2}\right)^{2k+1}$$

λ_n null derivation of Bessel function:

$$\lambda_1 = 1,841, \lambda_2 = 5,331, \lambda_3 = 8,536$$

ξ dimensionless radius: $\xi = r/R$

ζ dimensionless height: $\zeta = z/H$

θ angle of circumference

γ tank slenderness: $\gamma = H/R$

$a_{kn}(t)$ horizontal acceleration-time history as a result of an equivalent single-degree-of-freedom system with a period T_{kn} for the n^{th} -eigenmode of the sloshing wave. By using the response spectrum analysis the spectral accelerations corresponding to the natural periods T_{kn} should be calculated based on the elastic response spectrum.

Γ_{kn} participation factor for the convective pressure component for the n^{th} -eigenmode.

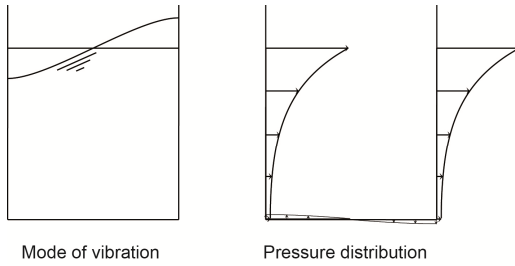


Fig. 2: Convective pressure - mode of vibration and pressure distribution

Taking into account the first sloshing eigenmode ($n = 1$) and the pressure distribution of the tank shell ($\xi = 1$), equation (1) can be simplified to:

$$p_k(\xi = 1, \zeta, \theta, t) = R \cdot \rho_L \left[0,837 \cdot \frac{\cosh(1,841 \cdot \gamma \cdot \zeta)}{\cosh(1,841 \cdot \gamma)} \right] \cdot [\cos(\theta)] [a_{k1}(t) \cdot \Gamma_{k1}] \quad (2)$$

The natural period T_{kn} for the n^{th} -eigenmode of the sloshing wave is calculated with:

$$T_{kn} = \frac{2\pi}{\sqrt{\frac{g \cdot \lambda_n \cdot \tanh(\lambda_n \cdot \gamma)}{R}}} \quad (3)$$

Rigid impulsive pressure

Figure 3 shows the mode of vibration and the pressure distribution corresponding to the rigid impulsive pressure component.

The pressure distribution is given by the expression:

$$p_{is,h}(\xi, \zeta, \theta, t) = \sum_{n=0}^{\infty} \frac{2 \cdot R \cdot \gamma \cdot \rho_L \cdot (-1)^n}{v_n^2} \left[\frac{l_1\left(\frac{v_n}{\gamma} \cdot \xi\right)}{l_1'\left(\frac{v_n}{\gamma}\right)} \right] \cdot [\cos(v_n \cdot \zeta)] [\cos(\theta)] [a_{is,h}(t) \cdot \Gamma_{is,h}] \quad (4)$$

with:

$p_{is,h}$ rigid impulsive pressure component due to horizontal excitation

$$v_n = \frac{2n+1}{2} \pi$$

l_1 modified first order Bessel function:

$$l_1\left(\frac{v_n}{\gamma} \cdot \xi\right) = \frac{J_1\left(i \cdot \frac{v_n}{\gamma} \cdot \xi\right)}{i^n}$$

$$= \sum_{k=0}^{\infty} \frac{1}{k! \cdot \Gamma(1+k+1)} \cdot \left(\frac{v_n}{\gamma} \cdot \xi\right)^{2k+1}$$

l_1' Derivation of the modified Bessel function regarding to DIN EN 1998-4 (2007)

$$l_1'\left(\frac{v_n}{\gamma} \cdot \xi\right) = l_0\left(\frac{v_n}{\gamma} \cdot \xi\right) - \frac{l_1\left(\frac{v_n}{\gamma} \cdot \xi\right)}{\left(\frac{v_n}{\gamma} \cdot \xi\right)}$$

$$l_1' = \sum_{k=0}^{\infty} \frac{1}{k! \cdot \Gamma(0+k+1)} \cdot \left(\frac{v_n}{\gamma} \cdot \xi\right)^{2k+0}$$

$$= \frac{\sum_{k=0}^{\infty} \frac{1}{k! \cdot \Gamma(1+k+1)} \cdot \left(\frac{v_n}{\gamma} \cdot \xi\right)^{2k+1}}{\frac{v_n}{\gamma} \cdot \xi}$$

$a_{is,h}(t)$ horizontal acceleration-time history. By using the response spectrum analysis $a_{is,h}(t)$ should be replaced by the spectral acceleration S_a corresponding to $T = 0$ s.

a_{gR} reference peak ground acceleration on type A ground

S soil factor

γ_i importance factor according to DIN EN 1998-1 (2010) or DIN EN 1998-4 (2007)

$\Gamma_{is,h}$ participation factor for the rigid impulsive pressure component: $\Gamma_{is,h} = 1,0$, because the rigid tank is moving together with the foundation.

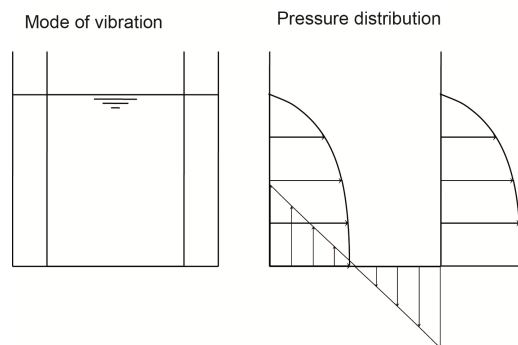


Fig. 3: Rigid impulsive pressure - Mode of vibration and pressure distribution

Taking into account the pressure distribution of the tank shell ($\xi = 1$), equation (4) is simplified to:

$$p_{is,h}(\xi = 1, \zeta, \theta, t) = R \cdot \rho_L \sum_{n=0}^{\infty} \left[\frac{2 \cdot \gamma \cdot (-1)^n}{v_n^2} \frac{l_1\left(\frac{v_n}{\gamma}\right)}{l_1'\left(\frac{v_n}{\gamma}\right)} \cos(v_n \cdot \zeta) \right] \cdot [\cos(\theta)] [a_{is,h}(t) \cdot \Gamma_{is,h}] \quad (5)$$

The corresponding natural period is $T = 0$.



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Flexible impulsive pressure

Figure 4 shows the mode of vibration and the pressure distribution corresponding to the flexible impulsive pressure component.

The flexible impulsive pressure component is calculated in an iterative procedure using the added-mass-model according to DIN EN 1998-4 (2007), Annex A. Within the framework of the procedure the tank wall is loaded with iterative calculated additional mass portions of the activated fluid. The pressure distribution is given by the expression:

$$p_{if,h}(\xi, \zeta, \theta, t) = \sum_{n=0}^{\infty} 2R \rho_L \left[\frac{I_1\left(\frac{v_n}{Y} \cdot \xi\right)}{\frac{v_n}{Y} \cdot I_1'\left(\frac{v_n}{Y}\right)} \right] \left[\cos(v_n \cdot \zeta) \int_0^1 f(\zeta) \cdot \cos(v_n \cdot \zeta) d\zeta \right] \cdot [\cos(\theta)] [a_{if,h}(t) \cdot \Gamma_{if,h}] \quad (6)$$

with:

$p_{if,h}$ flexible impulsive pressure component due to horizontal excitation

$f(\zeta)$ deflection curve of the first (anti-symmetric) mode of oscillation of the tank-fluid combination

$a_{if,h}(t)$ horizontal acceleration-time history as a result of an equivalent single-degree-of-freedom system. By using the response spectrum analysis the spectral accelerations corresponding to the first natural period $T_{if,h,1}$ should be used.

$\Gamma_{if,h}$ participation factor for the flexible impulsive pressure component due to horizontal excitation

The participation factor $\Gamma_{if,h}$ for the flexible impulsive pressure component is calculated as follows:

$$\Gamma_{if,h} = \frac{\int_0^1 p_{if,h}(\zeta) d\zeta}{\int_0^1 f(\zeta) \cdot p_{if,h}(\zeta) d\zeta}, \quad s(\zeta) = \text{const.} \quad (7)$$

with:

$p_{if,h}(\zeta)$ pressure function of the flexible impulsive pressure component as a function of the filling height

$s(\zeta)$ wall thickness of the tank

Taking into account the pressure distribution of the tank shell ($\xi = 1$), equation (6) can be simplified to:

$$p_{if,h}(\xi = 1, \zeta, \theta, t) = R \cdot \rho_L \sum_{n=0}^{\infty} \left[2 \cdot \frac{I_1\left(\frac{v_n}{Y}\right)}{\frac{v_n}{Y} \cdot I_1'\left(\frac{v_n}{Y}\right)} \cdot \cos(v_n \cdot \zeta) \int_0^1 f(\zeta) \cdot \cos(v_n \cdot \zeta) d\zeta \right] \cdot [\cos(\theta)] [a_{if,h}(t) \cdot \Gamma_{if,h}] \quad (8)$$

The first natural period $T_{if,h,1}$ is calculated as follows:

$$T_{if,h,1} = 2 \cdot F(Y) \sqrt{\frac{W_L}{\pi \cdot g \cdot E \cdot s(\zeta = 1/3)}} \quad (9)$$

$$= 2 \cdot R \cdot F(Y) \sqrt{\frac{H \cdot \rho_L}{E \cdot s(\zeta = 1/3)}}$$

with:

W_L fluid weight: $W_L = \pi \cdot R^2 \cdot H \cdot \rho_L \cdot g$

$F(Y)$ correction factor: $F(Y) = 0,157 \cdot Y^2 + Y + 1,49$
 $s(\zeta = 1/3)$ wall thickness of the tank at 1/3 filling height

Mode of vibration

Pressure distribution

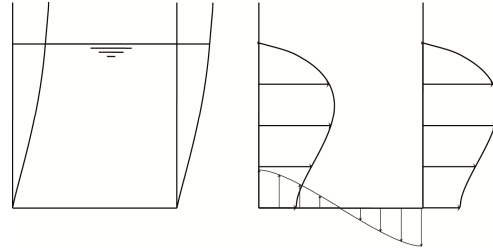


Fig. 4: Flexible impulsive pressure - Mode of vibration and pressure distribution.

The function curve of $f(\zeta)$ in (8) is generally not known. It depends on the impact of the liquid onto the tank, in other words the aforementioned pressure function $p_{if,h}$. Thus the joint bending form must be correctly determined iteratively. In DIN EN 1998-4 (2007), Annex A the "added-mass concept" is proposed. According to this the resonating fluid, activated with the first bending shape, is added to the tank wall density. Then with the new "dry" tank model, the more accurate bending form is determined (Fischer et. al., 1991).

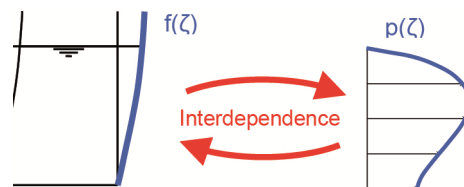
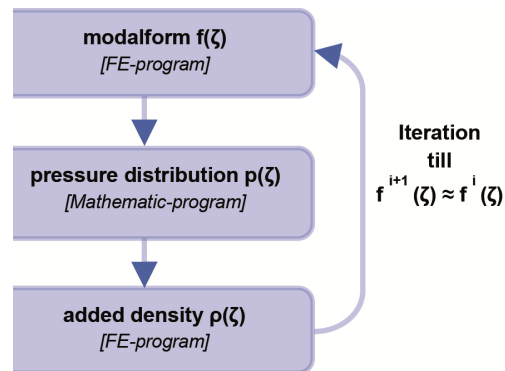


Fig. 5: Iteration process (Holtschoppen et. al., 2011)

However as Figure 5 shows, even this iterative process is impractical because the calculation is based on a complex mathematical pressure function (6) which requires a coupling of a mathematical software tool with a finite element program.

Though, with comprehensive parameter studies it is shown that the bending form of any tank can be described by using a parameterized sine wave which



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can be mapped correctly to the natural frequencies for all common geometric and material configurations (Cornelissen, 2010):

$$f(\zeta) = a \cdot \sin\left(\frac{\pi}{2} \cdot (\zeta - b) \cdot c + d\right) \quad (10)$$

The bending form of the combined vibration – and with this the parameters a , b , c , d – depends on

- the tank slenderness ($\gamma = H/R$),
- the Poisson's ratio ν ,
- the ratio fluid mass to tank mass and
- a changing wall thickness along the tank height.

However, the last named influences are comparatively small. By using the sine function (10) it is possible to determine the flexible impulsive pressure without iteration. For practical use and to represent the pressure the equation

$$p_{if,h}(\xi, \zeta, \theta, t) = R \cdot p_L \cdot \cos(\theta) \cdot \Gamma_{if,h} \cdot a_{if,h}(t) \cdot C_{if,h}(\xi, \zeta) \quad (11)$$

is suitable. $C_{if,h}(\xi, \zeta)$ corresponds to the normalized pressure at $\theta=0$. Figure 6 shows the variation of the factor $C_{if,h}(\xi=1, \zeta)$ for the tank shell ($\xi = 1$) for different tank slendernesses.

For simplicity, the formulations of all pressure functions are also provided in a standardized, tabulated form (Meskouris et. al., 2011).

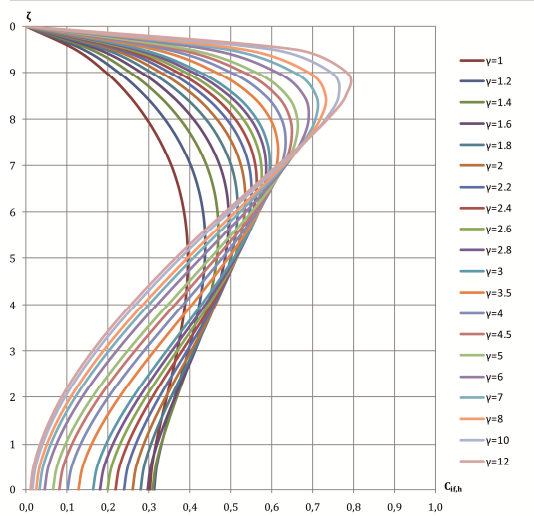


Fig. 6: Normalized flexible impulsive pressure $C_{if,h}$.

The participation factor for the flexible impulsive pressure $\Gamma_{if,h}$, which is specified in equation (7), can be tabulated as well, assuming that the tank wall is constant and the tank mass is insignificant.

CONCLUSION

This paper provides guidance for the implementation of normative demands for the seismic design of liquid filled tanks. With the tabulation of different factors the cumbersome mathematical formulas for calculating the load components are avoidable, which allows an easy load generation for finite element analysis.

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