LTB OF STEEL BEAMS WITH RESTRAINTS BETWEEN THE SUPPORTS

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INTRODUCTION

In Eurocode 3 [1] two methods to determine the ultimate lateral torsional buckling (LTB) load of steel beams in bending are presented: the General and the Specific Methods. These methods make use of buckling curves, but the curves to be used are different for each method. The ultimate lateral torsional buckling load can also be calculated using the Finite Element Method (FEM) by means of a geometrical and material non-linear analysis on a beam including imperfections (GMNIA). This paper compares the ultimate loads based on the design rules in Eurocode 3 [1] for lateral torsional buckling of laterally restrained beams in bending to the ultimate loads obtained with Finite Element (FE) simulations on the basis of a parameter study. One lateral restraint is applied between the supports. The location of the lateral restraint is varied along the length of the beam and along the height of the section. The stiffness of the lateral restraints is determined in such a way that the elastic critical moment is 95% of the elastic critical moment with rigid lateral restraints. A restraint having at least this stiffness is considered to be rigid. Three different load cases are considered: a simply supported beam loaded by a central point load, a simply supported beam subject to a uniformly distributed load and a propped cantilever beam with a central point load. It is concluded that the Specific Methods give a large over-estimation of the ultimate loads obtained from FEM simulations, while the General Method gives good results.

1 CODE REQUIREMENTS

For both the General and Specific Methods in Eurocode 3 [1] to determine the ultimate LTB load of beams in bending, the design buckling resistance moment should be taken as:

$$M_{b,Rd} = \chi_{LT} W_y f_y \tag{1}$$

in which W_y is the appropriate section modulus: $W_y = W_{pl,y}$ for class 1 or 2 sections. The reduction factor χ_{LT} is a function of the imperfection factor α_{LT} and the relative slenderness is given by:

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} \tag{2}$$

This relative slenderness will be used in subsequent equations to determine the reduction factor. It should be noted that the elastic critical bending moment for LTB, M_{cr} is not specified by Eurocode 3 [1]. Its determination is left to the designer.

1.1 General Method

This method is presented in clause 6.3.2.2 of Eurocode 3 [1] as 'general case', hereafter referred to as General Method (GM). According to the GM, the reduction factor $\chi_{LT,GM}$ for LTB of beams is analogous to the reduction factor for column buckling [2]:

$$\chi_{LT,GM} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \overline{\lambda}_{LT}^2}} \leq 1 \tag{3}$$

$$\Phi_{LT} = 0.5\left\{1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^2\right\} \tag{4}$$

When $\bar{\lambda}_{LT} \leq 0.4$ or the design bending moment $M_{Ed} \leq 0.16 M_{cr}$ then $\chi_{LT,GM} = 1$. The imperfection factor α_{LT} is selected according to the required buckling curve for the design of the beam. The appropriate buckling curve is given in Eurocode 3 [1], Table 6.4 and the pertaining imperfection factor can be found in Table 6.3. For a rolled section IPE240, buckling curve ‘a’ is specified.

1.1 Specific Method

The method as presented in clause 6.3.2.3 of Eurocode 3 [1] was developed specifically for LTB of rolled or equivalent sections and is here referred to as the Specific Method (SM). According to this method, the reduction factor $\chi_{LT,SM}$ is determined as follows:

$$\chi_{LT,SM} = \frac{1}{\Phi_{LT} + \sqrt{\Phi_{LT}^2 - \beta\bar{\lambda}_{LT}^2}} \leq 1 \text{ and } \leq \frac{1}{\bar{\lambda}_{LT}^2} \tag{5}$$

$$\Phi_{LT} = 0.5\left\{1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - \bar{\lambda}_{LT,0}\right) + \beta\bar{\lambda}_{LT}^2\right\} \tag{6}$$

in which $\bar{\lambda}_{LT,0} = 0.4$ (recommended maximum value) and $\beta = 0.75$ (recommended minimum value). The imperfection factor α_{LT} is selected according to the recommended buckling curve. These are given in Table 6.5 of Eurocode 3 [1]. For a rolled section IPE240, buckling curve ‘b’ is specified.
To take the moment distribution and the corresponding beneficial effect of reduced plastic zones [3] into account, the reduction factor may be modified by dividing $\chi_{LT,SM}$ by a factor *f*:

$$\chi_{LT,MSM} = \frac{\chi_{LT,SM}}{f} \tag{7}$$

$$f = 1 - 0.5\left(1 - k_c\right)\left\{1 - 2\left(\bar{\lambda}_{LT} - 0.8\right)^2\right\} \leq 1 \tag{8}$$

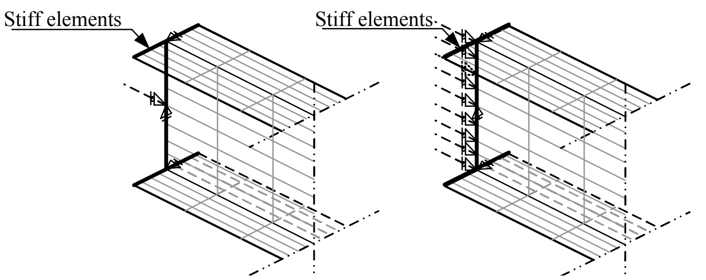
The correction factor k_c is given in Table 6.6 of Eurocode 3 [1]. In case the reduction factor is modified as indicated by *Eq. (7)*, the method is referred to as Modified Specific Method (MSM).

2 FINITE ELEMENT SIMULATIONS

FE simulations with the program ANSYS 10.0 using GMNIA for restrained beams yield ultimate LTB loads.

2.1 FE-model and loading

The FE model (*Figs. 1, 2*) of the restrained IPE beams consists mainly of 4 node shell elements based on Mindlin-Reissner shell theory. Using shell elements only would cause the root radii between flange and web to be ignored. To compensate for this, elastic-plastic rectangular hollow



(a) Simple support (b) Fixed support

Fig. 1. Support conditions

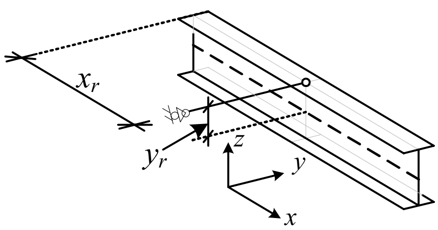


Fig. 2. Lateral restraint

section beam elements are used at the flange-web intersection [4].

Two different types of supports with torsion restraints yielding fork conditions are modelled: a simple support (*Fig. 1a*) and a fixed support where warping is not restrained (*Fig. 1b*). The fixed support is used for load case 3. The stiff (beam) elements (*Fig. 1*) are used to prevent distortion of the cross-section.

The lateral restraint is modelled by a single link element (*Fig. 2*). Its position varies along the longitudinal beam axis (x_r) and the section height (y_r). In case the lateral restraint is attached to the web, stiffeners have been introduced to prevent local distortion of the cross-section.

In the parameter study (section 3) three load cases will be investigated (*Fig. 3*). The concentrated load is applied to a single node requiring the introduction of stiffeners. The uniformly distributed load has been modelled by nodal loads applied at the intersection of the web and the upper flange.

2.2 Material and imperfections

The steel grade employed in all analyses was S235 with yield strength 235 N/mm^2. A bilinear stress-strain curve without hardening was used with a Young's modulus of elasticity of 210 kN/mm^2. The residual stresses were modelled as shown in *Fig. 4*. The maximum value of the residual stresses is taken to be one third of the yield stress [5].

According to Eurocode 3 [1], the first elastic critical buckling mode may be used to model the geometrical imperfections of the beam. If this is done, the imperfection pattern will vary from one simulation to the other depending on the location and the stiffness of the lateral restraint, leading to inconsistent results [4]. To avoid this, the imperfection pattern used is that of an unrestrained beam. An imperfection amplitude of $\ell/1000$ is used as suggested and used by other researchers (e.g. [6]) in case residual stresses are explicitly taken into account. This amplitude is applied at the location where the largest displacement is obtained in a linear buckling analysis (LBA). The first elastic critical buckling mode is scaled accordingly to obtain the imperfections.

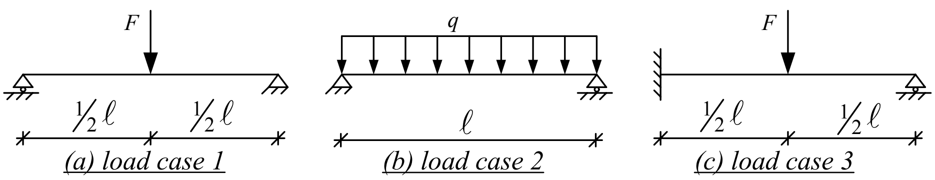


Fig. 3. Load cases

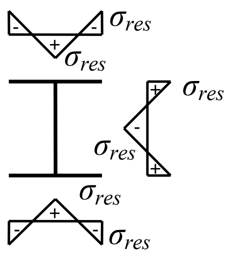


Fig. 4. Residual stress

3 PARAMETER STUDY

The parameter study is done for the three load cases (*Fig. 3*) on restrained IPE240 beams. The first case consists of a concentrated load at mid-span on a simply supported beam. The load is applied at the intersection of web and top flange. Three span lengths are considered: 7.2m, 5.4m and 3.6m. The second load case is a uniformly distributed load on a simply supported beam applied at the web top flange intersection. One span length is considered: 7.2m. The third load case is a statically indeterminate system. This load case resembles the first load case but here one of the supports is fixed such that warping is not restrained. For this load case again one span length is considered: 7.2m. The location of the lateral restraints is varied: six locations in longitudinal direction from support to mid span at regular intervals are considered and 5 locations from top flange centroid to bottom flange centroid at regular intervals are considered.

3.1 Position and stiffness of lateral restraint

In *Fig. 5* the influence of the location of an infinitely stiff lateral restraint on the elastic critical moment is shown for load case 1 with span length 7.2m. It is clear that a lateral restraint has more effect at the centre of the span and at the top flange, resulting in larger elastic critical moments.

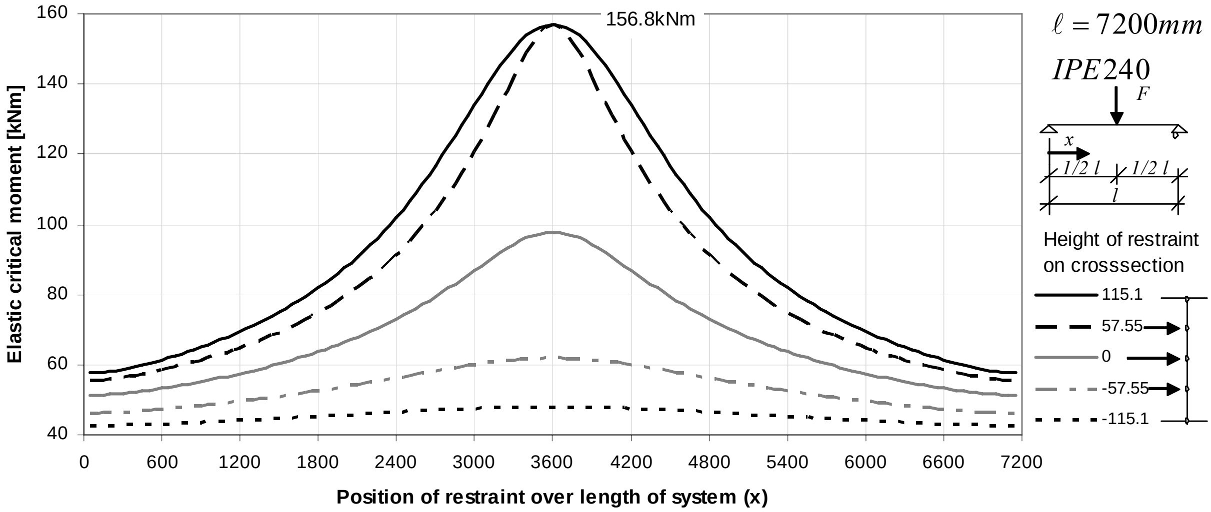


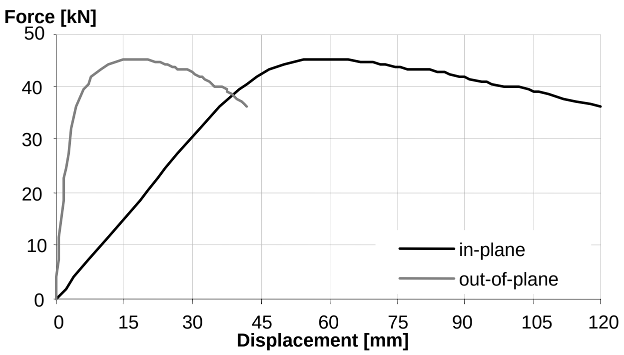
Fig. 5. Influence of restraint location on elastic critical moment

However, infinitely stiff lateral restraints do not exist in practice. Therefore, the stiffness of the restraint $K_{95\%}$ has been determined, allowing a maximum reduction of 5% in the elastic critical moment with infinitely stiff lateral restraints (as suggested in [7]).

3.2 Illustration of calculations

To illustrate the comparison between FE and EC3 suggested methods a simply supported steel beam, IPE240, with a length of 7200 mm is subjected to a concentrated load at mid-span, i.e. load case 1. The lateral restraint is located at mid-span at the centroid of the upper flange. The restraint stiffness $K_{95\%} = 438$ kN/m, determined by trial and error, is used. This is a relatively low value since the lateral support location is very effective. FE analyses for the elastic critical and ultimate loads in addition to the three methods of analysis, as described in Eurocode 3 [1] (section 1), result in four sets of values for relative slenderness $\bar{\lambda}_{LT}$ with reduction factor χ_{LT}.

The calculation of the relative slenderness requires prior knowledge of the elastic critical and design bending resistance moments. The design bending resistance moment of an IPE240 is $M_{pl} = W_y f_y = 86.151$ kNm. The elastic critical moment has been obtained from a FE linear buckling analysis (LBA) on a model as explained in section 2:



IPE240, load case 1,
$\ell = 7.2$ m, $K_{95\%} = 438$ kN/m

Fig. 6. Load-displacement curves

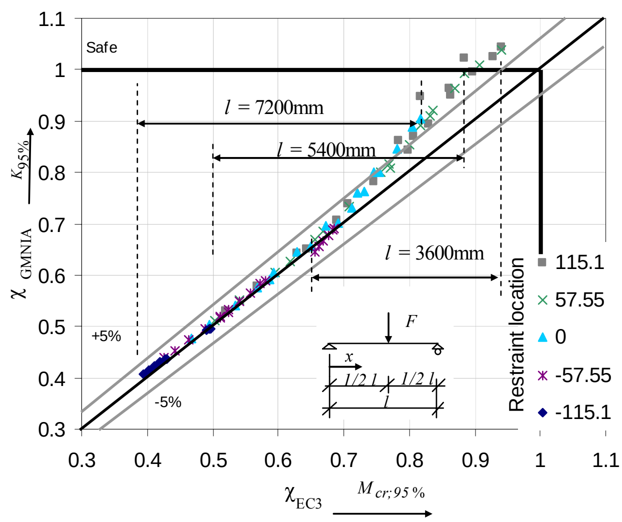


Fig. 7. GMNIA versus GM – load case 1

$M_{cr,95\%} = 148.990$ kNm. Using *Eq. (2)* results in a relative slenderness: $\bar{\lambda}_{LT,FEM} = 0.760$.

The application of a geometric and material non-linear analysis on an imperfect beam (GMNIA) gives a load-displacement curve as shown in *Fig. 6*, including the failure load of the beam being 45.317 kN. The bending moment at failure then becomes $M_u = 45.317 \times 7.2/4 = 81.571$ kNm. The reduction factor for the FEM can be determined from *Eq. (1)*: $\chi_{LT,FEM} = 0.947$.

The reduction factor to be used in the GM can be obtained using the earlier calculated relative slenderness of the beam, i.e. $\bar{\lambda}_{LT} = 0.760$. A value for Φ_{LT} can be determined from *Eq. (4)* for buckling curve 'a' where $\alpha_{LT} = 0.21$ for rolled sections: $\Phi_{LT,GM} = 0.848$. The reduction factor can now be calculated using *Eq. (3)*: $\chi_{LT,GM} = 0.818$.

The reduction factor to be used in the SM can also be obtained by employing the earlier calculated relative slenderness of the beam structure. The value for Φ_{LT} must now be determined from *Eq. (6)* for buckling curve 'b' where $\alpha_{LT} = 0.34$ for rolled sections: $\Phi_{LT,SM} = 0.778$. The reduction factor for this method comes from *Eq. (5)*: $\chi_{LT,SM} = 0.839$.

When the moment distribution is taken into account, the reduction factor can be increased. A modification factor can be obtained by using a correction factor $k_c = 0.86$. The modification factor is obtained from *Eq. (8)* which leads to: $f = 0.930$. The modified reduction factor that can be used in the MSM with moment distribution becomes with *Eq. (7)*: $\chi_{LT,MSM} = 0.901$.

3.3 Comparison

In *Figs. 7-9* comparisons between the reduction factors from the numerical simulations and those obtained by the GM are shown for load cases 1 to 3. The 45^o line indicates a perfect match between the results from FE simulations and the GM. The two lines next to the 45^o line mark a band width of 5% over- and underestimation. In these figures it is shown that the GM performs well.

In *Figs. 10, 11* comparisons between the reduction factors from the numerical simulations and those suggested by the SM and the MSM are shown for load case 1. It can be observed that results more than 10% on the unsafe side are present. The same holds for the other two load cases [4].

Similar results were obtained in [8] for lateral torsional buckling of unrestrained steel beams.

4 CONCLUSIONS

This paper compares ultimate lateral torsional buckling loads of restrained beams in bending based

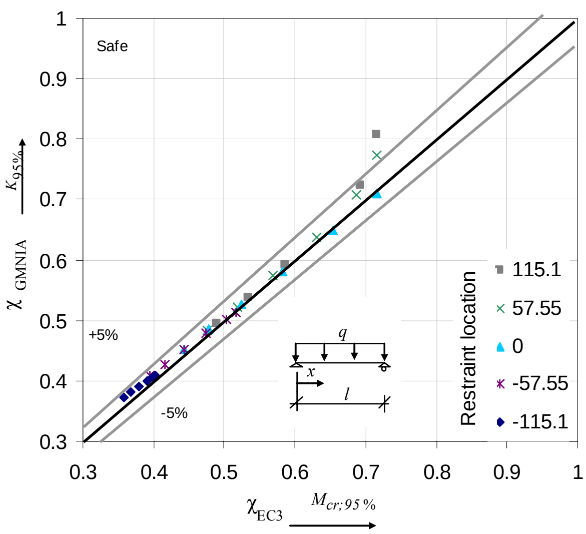


Fig. 8. GMNIA versus GM – load case 2

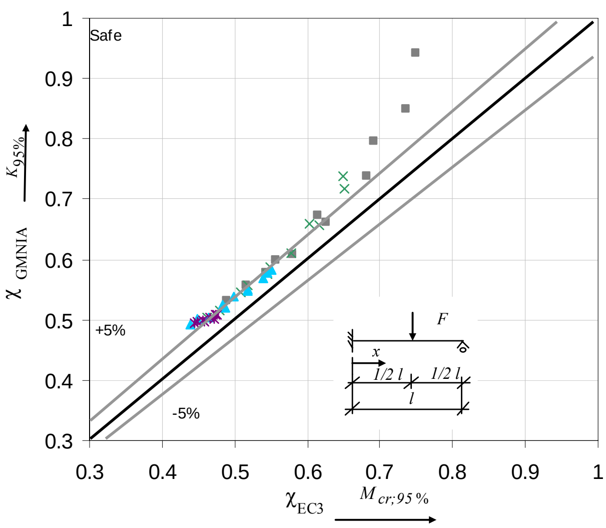


Fig. 9. GMNIA versus GM – load case 3

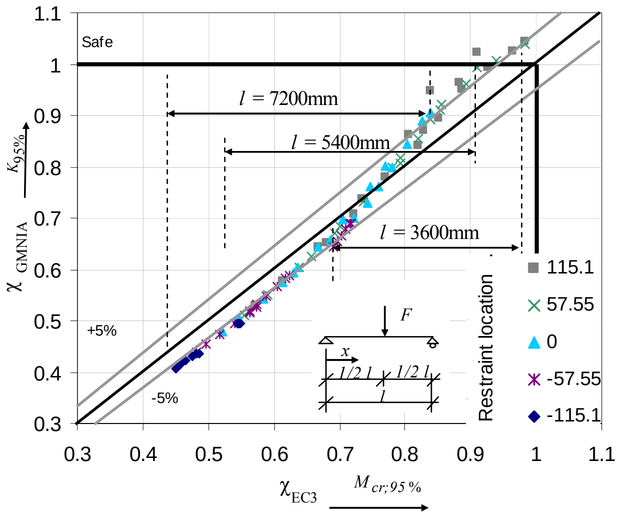


Fig. 10. GMNIA versus SM – load case 1

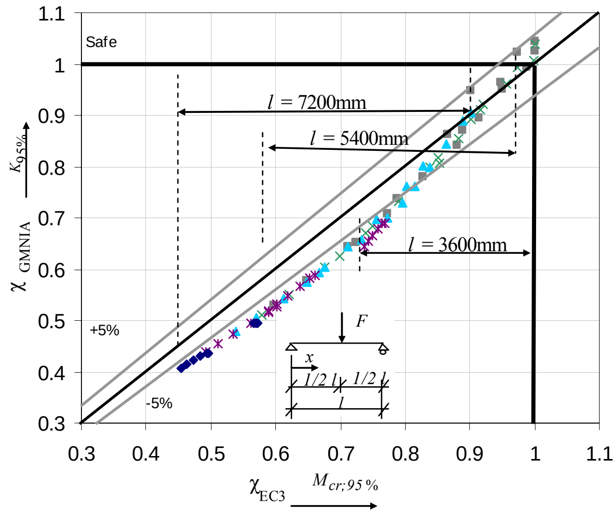


Fig. 11. GMNIA versus MSM – load case 1

on design rules in Eurocode 3 [1] to ultimate loads obtained with Finite Element simulations.
For the calculations performed in the parameter study, worrisome results have been obtained on the validity of the Specific Methods for lateral torsional buckling of rolled or equivalent welded sections. The Specific Methods give larger reduction factors than the General Method. It can be concluded that the Specific Methods can lead to large overestimations of even more than 10% of the ultimate lateral torsional buckling load of restrained beams obtained from Finite Element simulations. Therefore, Nationally Determined Parameters in the Specific Method should be chosen with care.
The General Method however gives good results for lateral torsional buckling of steel beams with restraints between the supports.

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